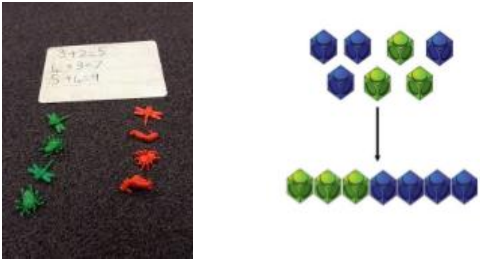
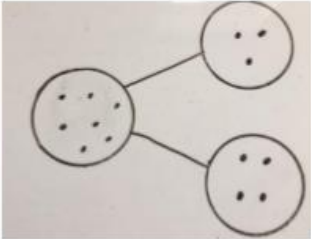
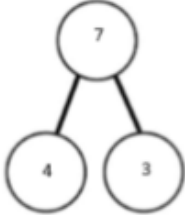


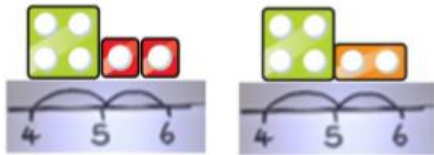
Calculation policy: Addition

Key language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same as'.

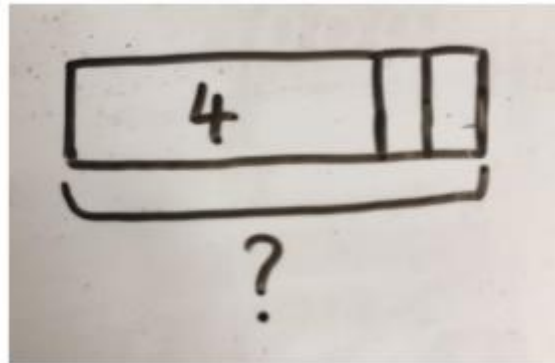
Use exchanging, or regrouping when moving into the next column.

Concrete	Pictorial	Abstract
<p>Combining two parts to make a whole – first by counting objects, then using mathematical equipment such as multilink or numicon.</p>  <p>The image shows a concrete representation of the addition 4 + 3 = 7. On the left, there are 4 green star-shaped objects and 3 red star-shaped objects. In the center, a small white card has the equation 4 + 3 = 7 written on it. On the right, there are 7 multilink cubes (4 blue and 3 green) arranged in a row. Above them, the same 4 blue and 3 green cubes are arranged in two groups, representing the two parts being added.</p>	<p>Children to represent the cubes using dots or crosses. They could put each part on a part whole model too.</p>  <p>The image shows a pictorial representation of the addition 4 + 3 = 7. On the left, there are 7 dots arranged in a circle. On the right, there are two smaller circles, each containing 3 dots. Lines connect the two smaller circles to the larger circle, representing a part-whole model where the whole is 7 and the parts are 4 and 3.</p>	<p>$4 + 3 = 7$ Four is a part, 3 is a part and the whole is seven.</p>  <p>The image shows an abstract representation of the addition 4 + 3 = 7. It features a large circle at the top containing the number 7. Two lines connect this circle to two smaller circles below it. The left smaller circle contains the number 4, and the right smaller circle contains the number 3.</p>

Counting on using number lines or numicon



A bar model that encourages children to count on.

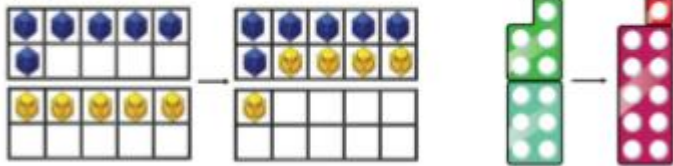


The abstract number line:
 What is 2 more than 4?
 What is the sum of 2 and 4?
 What is the total of 4 and 2?
 $4 + 2$

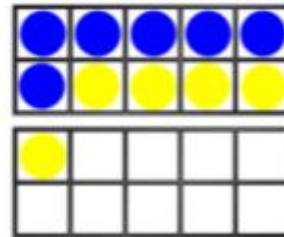


Regrouping to make 10; using ten frames and counters/cubes or using Numicon.

$6 + 5$



Children to draw the ten frame and counters/cube



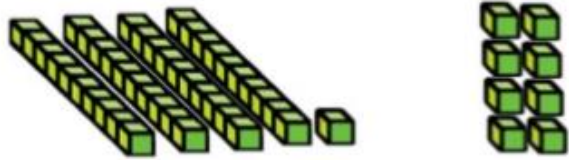
Children to develop an understanding of equality e.g.

$6 + \square = 11$

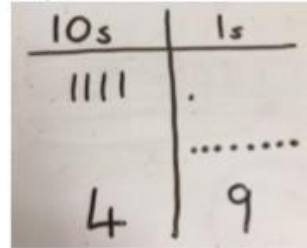
$6 + 5 = 5 + \square$

$6 + 5 = \square + 4$

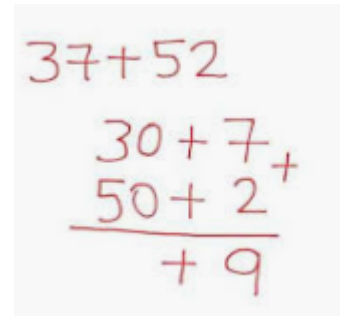
TO + O using base 10. Continue to develop understanding of partitioning and place value.
41 + 8



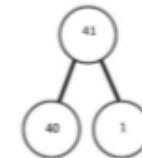
Children to represent the base 10 e.g. lines for tens and dot/crosses for ones.



Column method starting with expansion



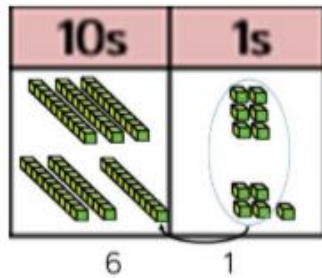
41 + 8



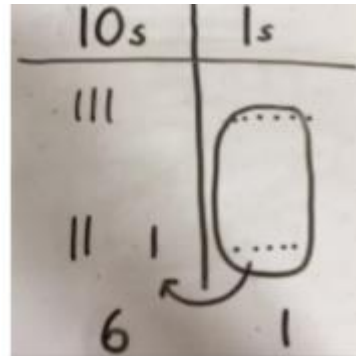
$$\begin{aligned} 1 + 8 &= 9 \\ 40 + 9 &= 49 \end{aligned}$$

	4	1
+		8
<hr/>		
	4	9

TO + TO using base 10. Continue to develop understanding of partitioning and place value.
 $36 + 25$



Children to represent the base ten in a place value chart.



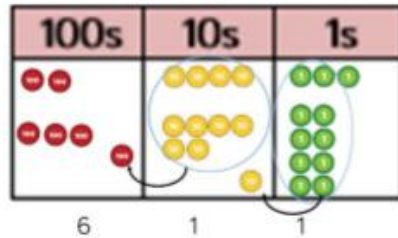
Looking for ways to make 10.

$$\begin{array}{r}
 36 + 25 = \\
 \swarrow \quad \searrow \\
 1 \quad 5
 \end{array}
 \begin{array}{l}
 30 + 20 = 50 \\
 5 + 5 = 10 \\
 50 + 10 + 1 = 61
 \end{array}$$

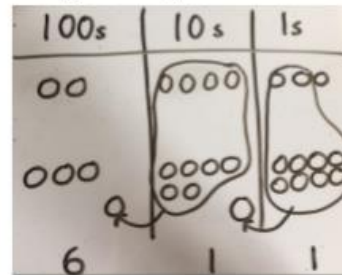
Formal method:

$$\begin{array}{r}
 +25 \\
 36 \\
 \hline
 61 \\
 \hline
 1
 \end{array}$$

Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.



Children to represent the counters in a place value chart, circling when they make an exchange.



Expanded method leading to column addition

$$353 + 268 = 621$$

$$300 + 50 + 3$$

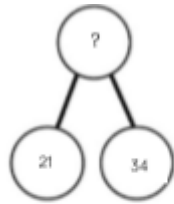
$$200 + 60 + 8$$

$$600 + 20 + 1 = 621$$

100 10

$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ \hline 11 \end{array}$$

Conceptual variation – different ways to ask children to add 21 + 34



?	
21	34

Word problems:
 In year 3, there are 21 children and in year 4, there are 34 children.
 How many children in total?

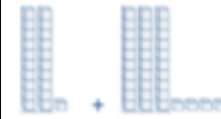
$21 + 34 = 55$. Prove it

$$\begin{array}{r} 21 \\ +34 \\ \hline \end{array}$$

$21 + 34 =$

 = $21 + 34$

Calculate the sum of twenty-one and thirty-four.

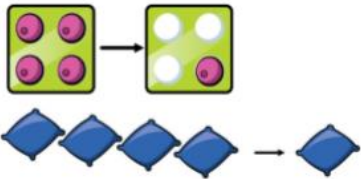
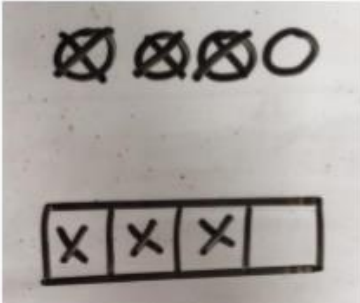
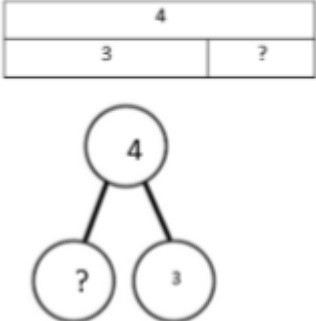
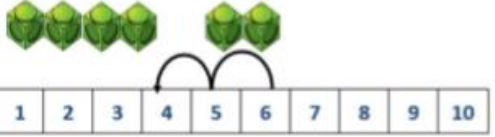
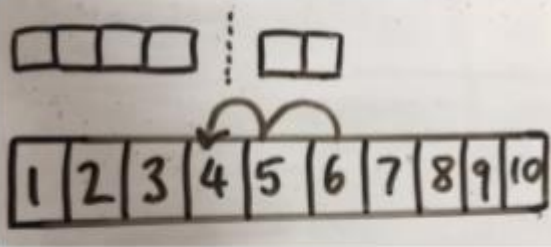
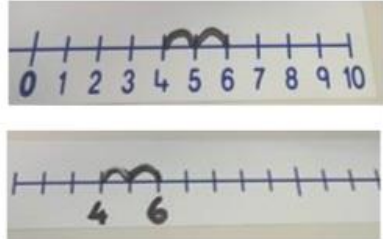


Missing digit problems:

10s	1s
● ●	●
● ● ●	?
?	5

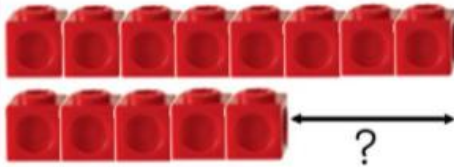
Calculation policy: Subtraction

Key language: take away, less than, the difference, subtract, minus, fewer, decrease.

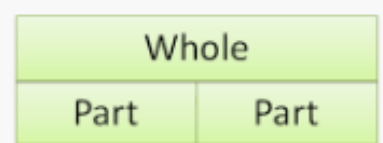
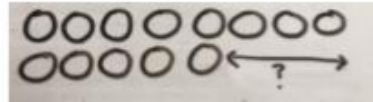
Concrete	Pictorial	Abstract
<p>Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used).</p> <p>$4 - 3 = 1$</p>  <p>Objects should be used first.</p>	<p>Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.</p> 	<p>$4 - 3 =$</p> <p>$\square = 4 - 3$</p> 
<p>Counting back (using number lines or number tracks) children start with 6 and count back 2.</p> <p>$6 - 2 = 4$</p> 	<p>Children to represent what they see pictorially</p> 	<p>Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line</p> 

Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used).

Calculate the difference between 8 and 5.



Children to draw the cubes/other concrete objects v they have used or use the bar model to illustrate wh they need to calculate.



$$\text{Part} + \text{Part} = \text{Whole}$$

$$\text{Whole} - \text{Part} = \text{Part}$$

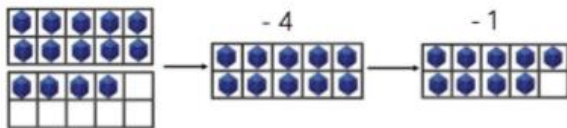
Find the difference between 8 and 5.

8 - 5, the difference is

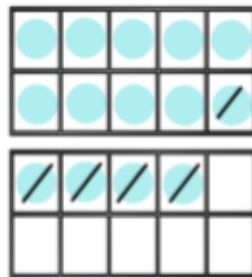
Children to explore why
 $9 - 6 = 8 - 5 = 7 - 4$ have the same difference.

Making 10 using ten frames.

14 - 5



Children to present the ten frame pictorially and what they did to make 10.

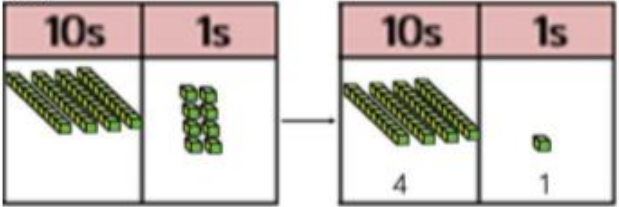
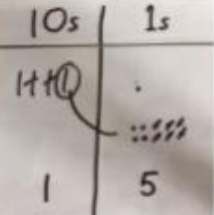
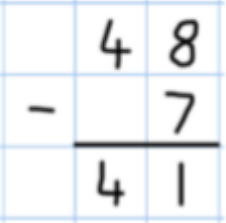
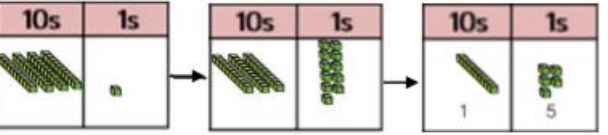
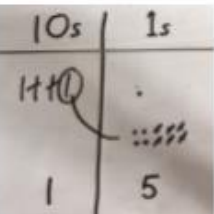
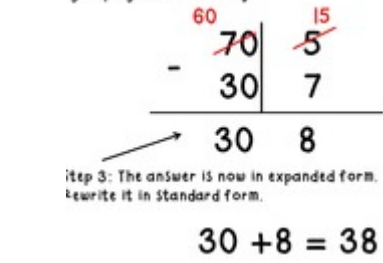
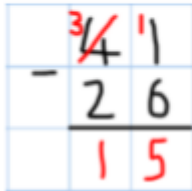


Children to show how they can make 10 by partitioning the subtrahend.

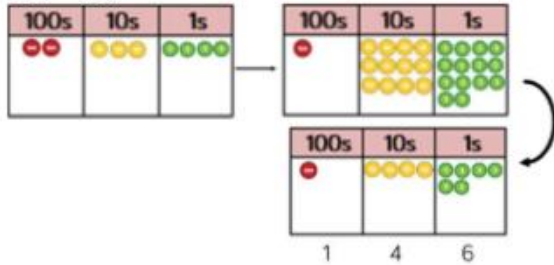
$$14 - 5 = 9$$

$$14 - 4 = 10$$

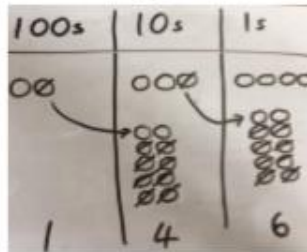
$$10 - 1 = 9$$

<p>Column method using base 10. 48-7</p> 	<p>Represent the base 10 pictorially, remembering to the exchange.</p> 	<p>Column method or children could count back 7.</p> 
<p>Column method using base 10 and having to exchange. 41 - 26</p> 	<p>Represent the base 10 pictorially, remembering to the exchange.</p> 	 <p>Step 3: The answer is now in expanded form. Rewrite it in Standard form.</p> $30 + 8 = 38$ <p>Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because $41 = 30 + 11$.</p> 

Column method using place value counters.
234 - 88



Represent the place value counters pictorially, remembering to show what has been exchanged



Column subtraction

$$942 - 214$$

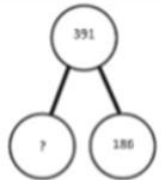
Expanded method

$$\begin{array}{r} 900 \quad 40 \quad 2 \\ - 200 \quad 10 \quad 4 \\ \hline 700 \quad 20 \quad 8 \end{array}$$

Compact Method

$$\begin{array}{r} 3 \quad 12 \\ 942 \\ - 214 \\ \hline 728 \end{array}$$

Conceptual variation; different ways to ask children to solve 391 - 186



391	
186	?

Raj spent £391, Timmy spent £186.
How much more did Raj spend?

Calculate the difference between 391 and 186.

$$\square = 391 - 186$$

$$\begin{array}{r} 391 \\ -186 \\ \hline \end{array}$$

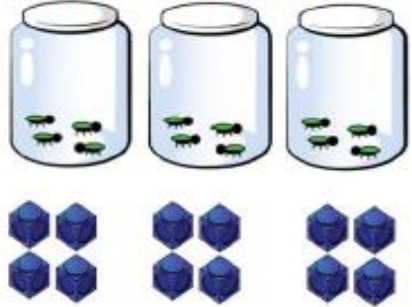
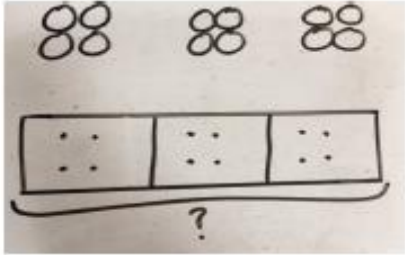

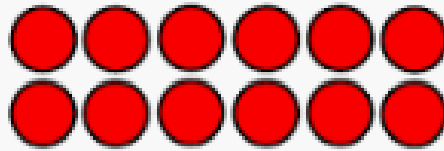
What is 186 less than 391?



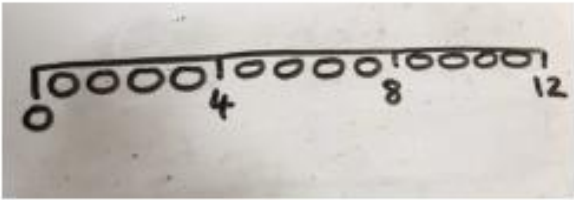
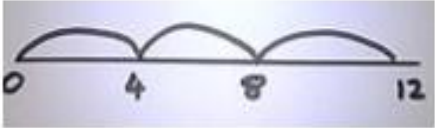


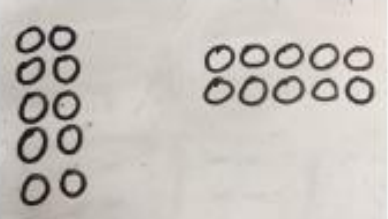
Missing digit calculations

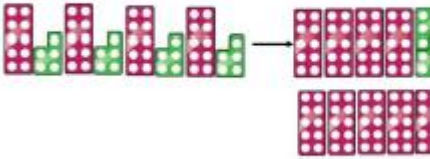
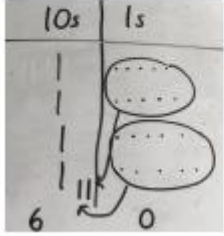
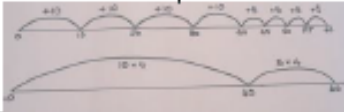
$$\begin{array}{r} 3 \quad 9 \quad \square \\ - \square \quad \square \quad 6 \\ \hline \square \quad 0 \quad 5 \end{array}$$

Calculation policy: Multiplication

Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups.

Concrete	Pictorial	Abstract
<p>Repeated grouping/repeated addition 3×4 $4 + 4 + 4$ There are 3 equal groups, with 4 in each group.</p>  <p>The concrete representation shows three jars, each containing four green beans. Below the jars are three groups of four blue blocks, each group arranged in a 2x2 square.</p>	<p>Children to represent the practical resources in a picture and use a bar model.</p>  <p>The pictorial representation shows three groups of two circles each. Below this is a bar model divided into three equal sections, each containing two dots. A bracket underneath the bar model is labeled with a question mark.</p>	<p>$3 \times 4 = 12$ $4 + 4 + 4 = 12$</p>
 <p>A photograph showing 12 cucumbers arranged in two rows of six.</p>	 <p>A pictorial representation of 2x6 using two rows of six red circles each.</p>	<p>$2 \times 6 = 12$ $6 \times 2 = 12$ Two lots of six is twelve. Six lots of two is twelve. $2 + 2 + 2 + 2 + 2 + 2 = 12$</p>

<p>Number lines to show repeated groups- 3×4</p>   <p>Cuisenaire rods can be used too.</p>	<p>Represent this pictorially alongside a number line e.g.:</p> 	<p>Abstract number line showing three jump of four.</p> $3 \times 4 = 12$ 	
<p>Use arrays to illustrate commutativity counters and other objects can also be used. $2 \times 5 = 5 \times 2$</p>  <p>2 lots of 5</p>  <p>5 lots of 2</p>	<p>Children to represent the arrays pictorially.</p> 	<p>Children to be able to use an array to w range of calculations e.g.</p> $10 = 2 \times 5$ $5 \times 2 = 10$ $2 + 2 + 2 + 2 + 2 = 10$ $10 = 5 + 5$	

<p>Partition to multiply using Numicon, base 10 rods.</p> <p>4×15</p> 	<p>Children to represent the concrete manipulatives pictorially.</p> 	<p>Children to be encouraged to show the steps they have taken.</p> $ \begin{array}{r} 4 \times 15 \\ \swarrow \searrow \\ 10 \quad 5 \end{array} $ <p> $10 \times 4 = 40$ $5 \times 4 = 20$ $40 + 20 = 60$ </p> <p>A number line can also be used</p> 																									
<p>Formal column method with place value counters (base 10 can also be used.) 3×23</p> <table border="1" data-bbox="208 703 454 887"> <thead> <tr> <th>10s</th> <th>1s</th> </tr> </thead> <tbody> <tr> <td>●●</td> <td>●●●</td> </tr> <tr> <td>●●</td> <td>●●●</td> </tr> <tr> <td>●●</td> <td>●●●</td> </tr> </tbody> </table> <p>6 9</p>	10s	1s	●●	●●●	●●	●●●	●●	●●●	<p>Children to represent the counters pictorially</p> <table border="1" data-bbox="692 655 943 898"> <thead> <tr> <th>10s</th> <th>1s</th> </tr> </thead> <tbody> <tr> <td>oo</td> <td>ooo</td> </tr> <tr> <td>oo</td> <td>ooo</td> </tr> <tr> <td>oo</td> <td>ooo</td> </tr> </tbody> </table> <p>6 9</p>	10s	1s	oo	ooo	oo	ooo	oo	ooo	<p>Then we introduce the grid method.</p> <p>$3 \times 23 =$</p> <table border="1" data-bbox="1178 727 1503 850"> <tr> <td>x</td> <td>20</td> <td>3</td> <td></td> </tr> <tr> <td>3</td> <td>60</td> <td>9</td> <td>=69</td> </tr> </table>		x	20	3		3	60	9	=69
10s	1s																										
●●	●●●																										
●●	●●●																										
●●	●●●																										
10s	1s																										
oo	ooo																										
oo	ooo																										
oo	ooo																										
x	20	3																									
3	60	9	=69																								
		<p>Children to record what it is they are doing to show understanding.</p> <p>3×23 $3 \times 20 = 60$</p> <p> $\begin{array}{r} 20 \quad 3 \\ 3 \times 3 = 9 \\ 60 + 9 = 69 \end{array}$ </p> <p>23</p> <p>$\begin{array}{r} \times 3 \\ \hline 69 \end{array}$</p>																									

<p>Formal column method with place value counters. 6 x 23</p>	<p>Children to represent the counters/base 10, pictorially e.g. the image below.</p>	<p>Use grid method to show 6 lots of 20 and 6 lots of 3</p>	<p>Expanded</p>	<p>Formal written method</p> $6 \times 23 =$ $\begin{array}{r} 23 \\ \times 6 \\ \hline 18 \\ + 120 \\ \hline 138 \end{array}$								
<p>Multiplying 2-digit by 2-digit etc. Only move on to these type of calculations when children are confident and show understanding of the grid method. Teach understanding of adding the zero in the formal written method by saying that 20 is 10x larger than 2, so you need to add the zero to make the product 10x larger.</p>	<table border="1" data-bbox="1106 667 1429 863"> <tr> <td>x</td> <td>20</td> <td>3</td> </tr> <tr> <td>70</td> <td>1400</td> <td>210</td> </tr> <tr> <td>4</td> <td>80</td> <td>12</td> </tr> </table> <p>Then add together</p>	x	20	3	70	1400	210	4	80	12	<p>Expanded</p>	<p>Formal written method</p>
x	20	3										
70	1400	210										
4	80	12										
<p>Multiplying with decimals – disregard the decimals to do the multiplication, and then insert the decimal point into the product. Match up the amount of decimal places in the multiplicands and the product as shown.</p>	<p>So, $3.1 \times 5.9 = 18.29$</p>											

Conceptual variation; different ways to ask children to solve 6×23

23	23	23	23	23	23
----	----	----	----	----	----

?

Mai had to swim 23 lengths, 6 times a week.
How many lengths did she swim in one week?

With the counters, prove that $6 \times 23 = 138$

Find the product of 6 and 23

$$6 \times 23 =$$

$$\square = 6 \times 23$$

$$\begin{array}{r} 6 \\ \times 23 \\ \hline \end{array} \quad \begin{array}{r} 23 \\ \times 6 \\ \hline \end{array}$$

What is the calculation?
What is the product?

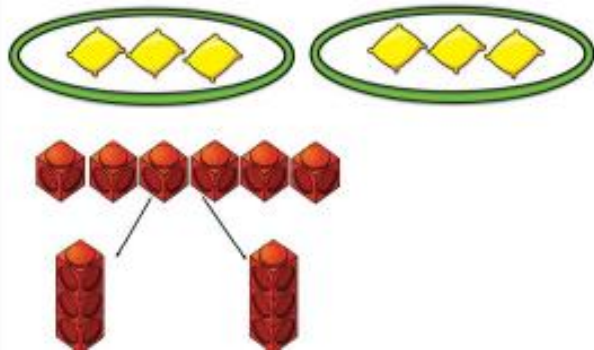
100s	10s	1s
		

Calculation policy: Division

Key language: share, group, divide, divided by, half.

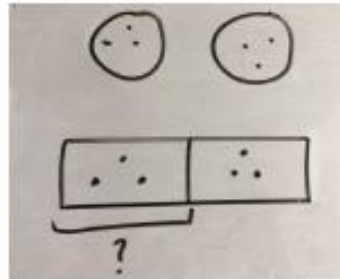
Concrete

Sharing using a range of objects.
 $6 \div 2$



Pictorial

Represent the sharing pictorially.

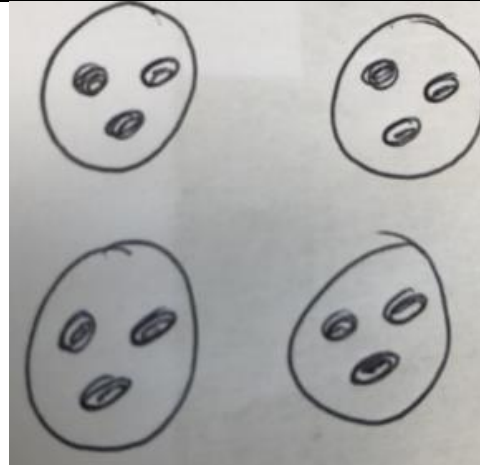


Abstract

$6 \div 2 = 3$

3	3
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Children should also be encouraged to use their 2 times tables facts.

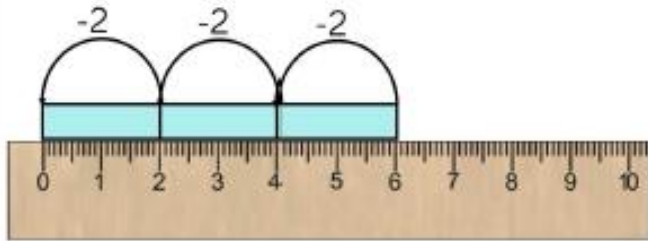


If you had 12 sweets and shared them between 4 children, how many would they get each?

12 shared between 4 is 3 each.

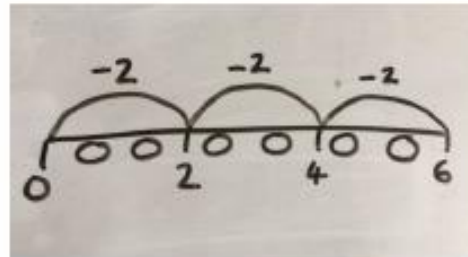
12 divided by 4 is 3.

Repeated subtraction using Cuisenaire rods above a ruler.
6 - 2

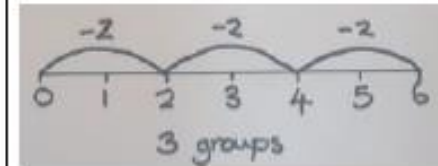


3 groups of 2

Children to represent repeated subtraction pictorially.



Abstract number line to represent the equal groups that have been subtracted.



2d + 1d with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used.

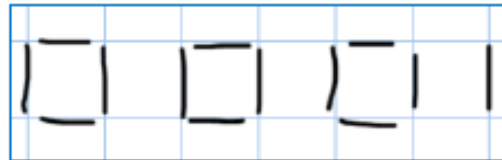
$13 \div 4$

Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.



There are 3 whole squares, with 1 left over.

Children to represent the lollipop sticks pictorially.

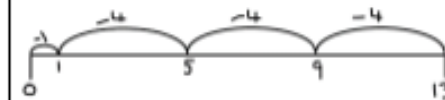


There are 3 whole squares, with 1 left over.

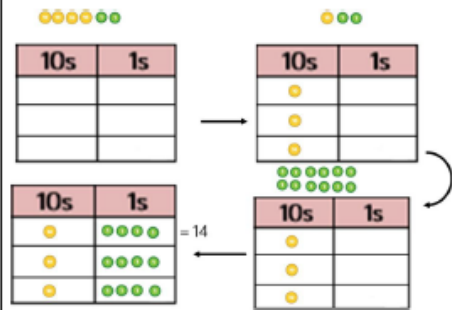
$13 \div 4 = 3 \text{ remainder } 1$

Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line.

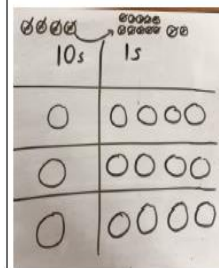
'3 groups of 4, with 1 left over'



Sharing using place value counters.
 $42 \div 3 = 14$



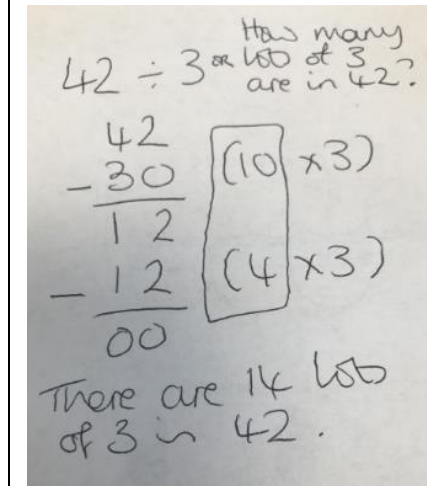
Children to represent the place value counters pictorially.



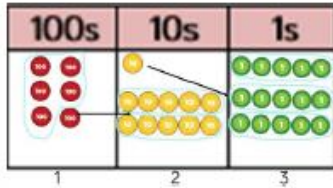
Children to be able to make sense of the place value counters and write calculations to show the process.

$$\begin{aligned} 42 \div 3 \\ 42 = 30 + 12 \\ 30 \div 3 = 10 \\ 12 \div 3 = 4 \\ 10 + 4 = 14 \end{aligned}$$

Teach chunking as an important conceptual step - link 'lots of' with division and multiplication.

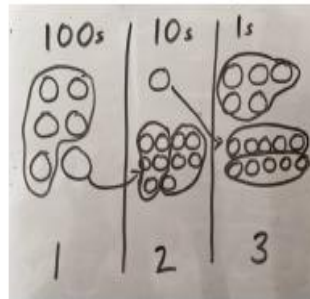


Short division using place value counters to group.
 $615 \div 5$

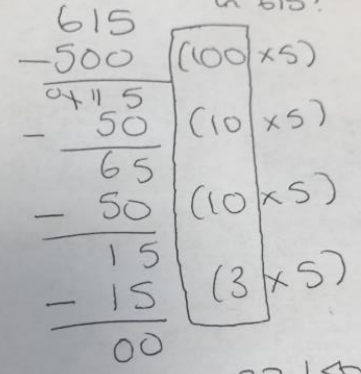


1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Represent the place value counters pictorially.



$615 \div 5$ or How many lots of 5 are in 615?



There are 123 lots of 5 in 615.

Children to the calculation using the short division scaffold.

$$5 \overline{) 615}$$

123

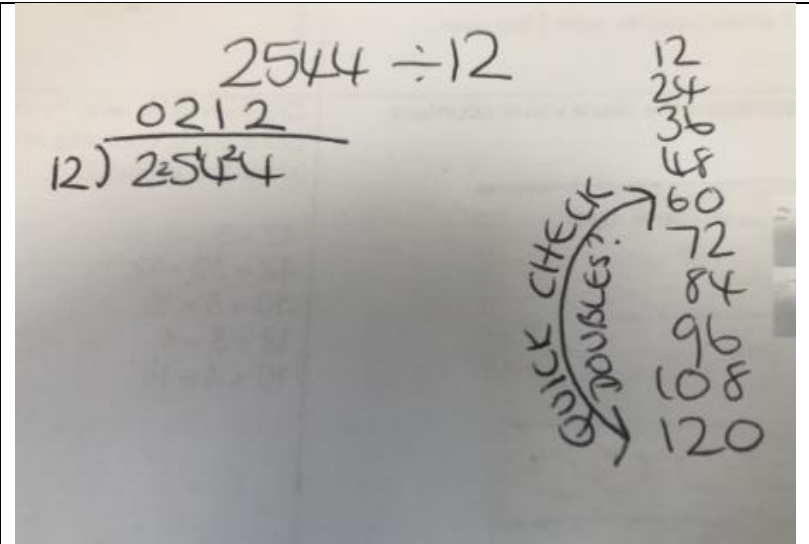
Long division using place value counters
 $2544 \div 12$

1000s	100s	10s	1s
●●	●●●●●●●●	●●●●●●●●	●●●●●●●●

We can't group 2 thousands into groups of 12 so will exchange them.

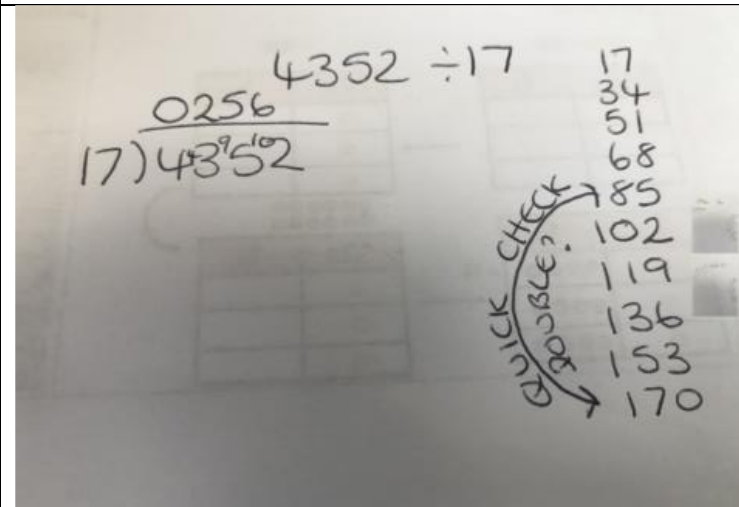
1000s	100s	10s	1s
	●●●●●●●●●●●●●●	●●●●●●●●	●●●●●●●●

We can group 24 hundreds into groups of 12 which leaves with 1 hundred.



Encourage writing out of factors using repeated addition before attempting the long division calculation.

Make sure children leave enough space to carry over digits within the calculation.



Division with remainders expressed as a decimal.

$$468 \div 5$$

$$\begin{array}{r} 093.6 \\ 5 \overline{) 468.30} \end{array}$$

Add a decimal point to the dividend and the quotient and as many zeroes to the dividend as needed

Decimals with remainders expressed as a fraction.

$$468 \div 5$$

$$\begin{array}{r} 093 \frac{3}{5} \\ 5 \overline{) 468} \end{array}$$

Numerator is the remainder, denominator is the divisor.

Conceptual variation; different ways to ask children to solve $615 \div 5$

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

$$5 \overline{) 615}$$

$615 \div 5 =$
 $\square = 615 \div 5$

What is the calculation?
What is the answer?

100s	10s	1s